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A Study of Actions On Multiplicative Neutrosophic Groups

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Abstract

The objective of this paper is to study the concept of algebraic action on neutrosophic group.

In this work we define weak neutrosophic actions and strong neutrosophic actions over a multiplicative neutrosophic group N(G), and presents some of their elementary properties.

Key words: Weak Neutrosophic action, Strong neutrosophic action, Multiplicative neutrosophic group, Indeterminacy.

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دراسة العمليات على الزمر النتروسوفية الضربية

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الملخّص

الهدف من هذا المقال هو دراسة مفهوم العملية الجبرية على الزمر النتروسوفية .هذا البحث يعرف مفهوم العملية النتروسوفية الضعيفة والعملية النتروسوفية القوية على زمرة نتروسوفية ضربية بالاضافة الى تقديم العديد من الخواص الاساسية لهذه المفاهيم .

الكلمات المفتاحية : عملية نيوتروسوفية ضعيفة ، عملية نيوتروسوفية قوية ، مجموعة نيوتروسوفية مضاعفة .لا حتمية .

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1. Introduction

Neutrosophy is a new generalized logic deals with indeterminacy in all the fields of knowledge. It was presented by the American mathematician F.Smarandache in 1995 as a generalization of Fuzzy logic [1].

The algebraic theory of neutrosophy began with the great efforts of Kandasamy and Smarandache, where the proposed concept of neutrosophic group and neutrosophic ring in [12].

Recently, there is an increasing interesting in neutrosophic algebraic structures such as neutrosophic ideals [1,2,17], homomorphisms [8,18,23,25], equations, and number theory [10,27], spaces [3,4,16,21,22], and modules [9,13,14].

Neutrosophic groups were studied widely, especially their substructures such as AH-solvability [7,15], and *n*-refined nilpotency, normality [11] and graphs [5].

Through this paper, we concentrate on defining actions over this kind of groups as a generalization of classical actions on classical groups. In particular, we present many interesting properties of these actions.

2. Preliminaries

Definition 2.1 :[7]

Let $(G,*)$ be a multiplicative group, then the multiplicative neutrosophic group is generated by G and I under * denoted by $N(G) = \{< G \cup I >,*\}$

is called the indeterminate element (neutrosophic element) with the property $I^2 = I$.

The most useful understanding of this definition had been written in [2] , where we consider *N(G)* as a union of *G* and *GI*, i.e. *N(G)*=$\{x_1, x_2, \dots x_1I, x_2I, \dots.; x_i \in G\}$.

.Definition 2.2 :[7]

Let *N(G)* be a neutrosophic group and H be a neutrosophic subgroup, i.e. (*H* contains a proper subgroup of G) of *N(G)*, then *H* is a neutrosophic normal subgroup of *N(G)* if $xH = Hx$ for all $x \in N(G)$.

Definition 2.3 :[5]

Let *N(G)* be a neutrosophic group. The center of *N(G)* is denoted by *C(N(G))* and defined as

$C\big(N(G)\big) = \{x \in N(G); xy = yx \ \ \forall\, y \in N(G)\}$.

Definition 2.4 : [7]

Let N(G) , N(H) be two neutrosophic groups, then

$. N(G) \times N(H) = \{(g,h); g \in N(G), h \in N(H)\}$

Definition 2.5 :[5]

Let $N(G)\,, N(H)$ be two neutrosophic groups and $\varphi: N(G) \to N(H)$ is called a neutrosophic homomorphism if it is a homomrphism between $G\,, H$, and $\varphi(I) = I'$

where I' is the neutrosophic element of $N(H)$.

If φ is a correspondence one-to-one then it is called a neutrosophic isomorphism.

Theorem 2.1 :[5]

$G \cong H \ \ if \ and \ \ only\, if \ \ N(G) \cong N(H)$.

Example 2.1 :

.Let *G*=($\{1,-1\}$,.) , we must understand *N(G)*=$\{\ 1\ ,-1\ ,I\ ,-I\ \}$

Theorem 2.2 :[5]

If G is a finite group, then $O(N(G)) = 2\ O(G)$.

Definition 2.6 : [6]

Let G be a group and A be any non empty set, then the well defined map $*: G \times A \to A$ is called an action if

$e * a = a$ and $x * (y * a) = (x\,y) * a$ for all $a \in A, x, y \in G$.

Definition 2.7 : [6]

$G_a = \{ x \in G ; x * a = a\}$, is called the stabilizer of *a* and it is a subgroup of *G*.

Remark 2. 1: [6]

If H is a subgroup of *G*, then there is a group homomorphism $f: G \to S_A$ with $\ker f \leq H$.

3. Actions on neutrosophic groups

Definition 3.1:

Let *N(G)* be a neutrosophic group and A be a non empty set with a fixed element J, let $*: N(G) \times A \to$ A be a well defined map , we say that (*) is a strong neutrosophic action if the following conditions hold:

(a) $e * a = a, I * a = a$.

(b) $x * (y * a) = (x\,y) * a\ for\ all\ a \in A, x, y \in N(G)$.

We call (*) a weak neutrosophic action if

(a) $e * a = a, I * J = J$

(b) $x * (y * a) = (x\,y) * a\ for\ all\ a \in A, x, y \in N(G)$.

It is clear that every strong action is a weak one.

We call the neutrosophic action (*) transitive if for every $a, b \in A\ \exists\ g \in N(G)$ such that $g * a = b$.

Examples 3.1 :

(a) Every neutrosophic group *N(G)* has a weak action on itself defined by

$*: N(G) \times N(G) \to N(G)$ with $x * y = x\,y\ for\ all\ x, y \in N(G)$.

(b) Let $H(I)$ be a neutrosophic subgroup of *N(G)*, then it acts strongly on *N(G)* by

$*: H(I) \times N(G) \to N(G)$ with $h * g = hg\ for\ all\ h \in H\ and\ g \in G$.

Theorem 3.1 :

If the group *G* acts on the set *A* by *, then the neutrosophic group *N(G)* acts strongly on *A*.

Proof :

Let (.) $: N(G) \times A(I) \to$A(I) with $g\,.\,a = g * a\ if\ g \in G$ and if $g = xI \in GI$ then $xI\,.\,a = x * a$.

It is easy to see that the previous map is a strong action because $I * a = a\ for\ all\ a \in A$.

Theorem 3.2 :

If the neutrosophic group *N(G)* acts on *A* by strong or weak action, then *G* acts on *A*.

Proof :

It is easy to see that the action (*) defined above can be restricted on *G*×*A*.

Theorem 3.3 :

Let *N(G)* be a neutrosophic group acts strongly on A, then the relation ^ defined as $a \,^\wedge b$ *if and only if there is* $g \in N(G)$, then $g * a = b$ is a correspondence relation.

Proof :

We have $a \,^\wedge a$ because $e * a = a$, and if we have $a \,^\wedge b$ and $b \,^\wedge c$, then there are $x, y \in N(G)$ such that

$(x * a = b, y * b = c)$, hence $y * (x * a) = c$, this implies $(y x) * a = c$ so that $a \,^\wedge c$.

Now, we assume that $a \,^\wedge b$, then there exists $x \in N(G)$ such that $x * a = b$. If $x \in G$ *then* $a = x^{-1} * b$.

If $x = y I \in GI$, then $yI * a = b$ so that $I * a = y^{-1} * b = a$ and $b \,^\wedge a$ clearly.

Theorem 3.4 :

Let *N(G)* be a neutrosophic group acts on A by a weak action. The relation ^ defined as

($a^\wedge b$ if and only if there is g $\in N(G)$ then $g * a = b$) is reflexive, transitive with the property $a \,^\wedge b$ implies $b \,^\wedge (I * a)$ *or* $b \,^\wedge a$.

proof :

We have $a \,^\wedge a$ because $e * a = a$, and if we have $a \,^\wedge b$ and $b \,^\wedge c$ then there are $x, y \in N(G)$ such that

$(x * a = b, y * b = c)$ *so that* $y * (x * a) = c$ *thus* $(y x) * a = c$ *so* $a \,^\wedge c$.

Now assume that $a \,^\wedge b$ then there is $x \in N(G)$ such $x * a = b$, if $x \in G$, then $a = x^{-1} * b$.

If $x = y I \in GI$, then $yI * a = b$, so that $I * a = y^{-1} * b$ so the proof is complete.

Definition 3.2 :

Let N(G) be a neutrosophic group acts on A , for $x \in A$ we define

$Orb(x) = \{ g * x ; g \in N(G) \}$.

The stabilizer $G_a = \{ x \in N(G) ; x * a = a \}$.

The kernel of (*) is defined as $\bigcap_{a \in A} G_a$.

Theorem 3.5:

Let *N(G)* be a neutrosophic group acts on *A* by strong or weak action, for any $a \in A$, we have G_a is a neutrosophic subgroup of *N(G).*

Proof :

By Theorem 3.4, we find that G_a contains the stabilizer subgroup of *a* in G, thus G_a is a neutrosophic subgroup.

Remark 3.1:

If (*) is a simple neutrosophic action, then it is clear that $I \in G_a$ for all $a \in A$.

Theorem 3.6 :

Let *N(G)* be a neutrosophic group and *H(I)* be a neutrosophic subgroup of *N(G),* where *H* is a subgroup of *G*, then there is a neutrosophic homomorphism

$\varphi : N(G) \to N(S_A)$ with N(Ker φ) $\leq H(I)$.

proof :

G acts on *A = { gH ; ∈ G }* by (*) defined *as* $g * (aH) = (ga) H$, hence there is a group homomorphism $f: G \to S_A$ with $ker\, f \leq H$. This means that we can find a neutrosophic homomorphism $\varphi : N(G) \to N(S_A)$ with $\varphi_G = f$ and $N(Ker\, \varphi) \leq H(I)$.

Conclusion

In this paper, the concept of algebraic action over multiplicative neutrosophic groups is studied Many properties were proved in a similar way to the classical concept of actions.

As a future research direction, this work can be extended into refined and *n*-refined neutrosophic groups respectively, where actions on these kinds of generalized structures are still unknown.

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